Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Further Pure Mathematics FP3

Advanced/Advanced Subsidiary

Monday 25 June 2018 – Morning
Time: 1 hour 30 minutes

Paper Reference
6669/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

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1. (a) Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, show that, for $x \in \mathbb{R}$

$$\tanh x = \frac{e^{2x} - 1}{e^{2x} + 1}$$

(2)

(b) Hence, given that $-1 < \theta < 1$, prove that

$$\operatorname{artanh} \theta = \frac{1}{2} \ln\left(\frac{1+\theta}{1-\theta}\right)$$

(3)

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Question 1 continued

Q1

(Total 5 marks)

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2.

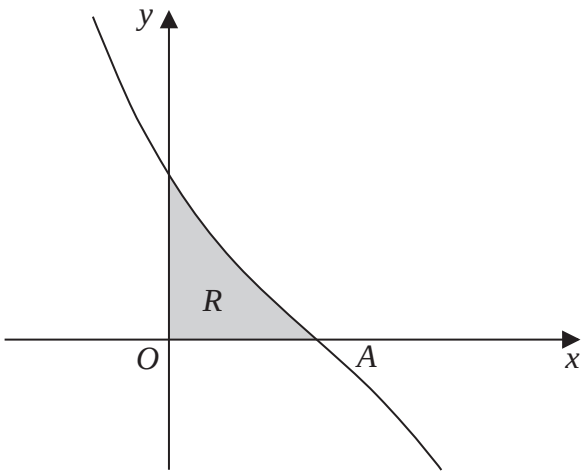

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 5\cosh x - 6\sinh x$$

The curve crosses the x-axis at the point A.

(a) Find the exact value of the x coordinate of the point A, giving your answer as a natural logarithm.

(3)

(b) Show that

$$(5\cosh x - 6\sinh x)^2 \equiv a\cosh 2x + b\sinh 2x + c$$

where a, b and c are constants to be found.

(3)

The finite region R, bounded by the curve and the coordinate axes, is shown shaded in Figure 1.

The region R is rotated through 2π radians about the x-axis.

(c) Use calculus to find the volume of the solid generated, giving your answer as an exact multiple of π.

(4)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total 10 marks)

Q2

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3. $\mathbf{M} = \begin{pmatrix} 3 & k & 2 \\ -1 & 0 & 1 \\ 1 & k & 1 \end{pmatrix}$, where k is a constant

Given that 3 is an eigenvalue of **M**,

(a) find the value of k.

(3)

(b) Hence find the other two eigenvalues of **M**.

(4)

(c) Find an eigenvector corresponding to the eigenvalue 3

(2)

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Question 3 continued

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Question 3 continued

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Question 3 continued

Q3

(Total 9 marks)

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4. The curve C has equation

$$y = \operatorname{arsinh} x + x\sqrt{x^2 + 1}, \qquad 0 \leqslant x \leqslant 1$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2\sqrt{x^2 + 1}$

(4)

(b) Hence show that the length of the curve C is given by

$$\int_0^1 \sqrt{4x^2 + 5}\ \mathrm{d}x$$

(2)

(c) Using the substitution $x = \dfrac{\sqrt{5}}{2}\sinh u$, find the exact length of the curve C, giving your answer in the form $a + b\ln c$, where a, b and c are constants to be found.

(6)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total 12 marks)

Q4

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5. Given that

$$I_n = \int x^n \sqrt{(x+8)}\ \mathrm{d}x, \qquad n \geqslant 0, x \geqslant 0$$

(a) show that, for $n \geqslant 1$

$$I_n = \frac{px^n(x+8)^{\frac{3}{2}}}{2n+3} - \frac{qn}{2n+3} I_{n-1}$$

where p and q are constants to be found.

(6)

(b) Use part (a) to find the exact value of

$$\int_0^{10} x^2 \sqrt{(x+8)}\ \mathrm{d}x$$

giving your answer in the form $k\sqrt{2}$, where k is rational.

(5)

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Question 5 continued

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Question 5 continued

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Question 5 continued

Q5

(Total 11 marks)

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6. The line l_1 has equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$$

where λ is a scalar parameter.

The line l_2 has equation

$$\frac{x+1}{1} = \frac{y-4}{1} = \frac{z-1}{3}$$

(a) Prove that the lines l_1 and l_2 are skew.

(4)

(b) Find the shortest distance between the lines l_1 and l_2

(5)

The plane Π contains l_1 and intersects l_2 at the point (3, 8, 13).

(c) Find a cartesian equation for the plane Π.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total 13 marks)

Q6

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7. The ellipse E has foci at the points $(\pm 3, 0)$ and has directrices with equations $x = \pm \frac{25}{3}$

(a) Find a cartesian equation for the ellipse E.

(5)

The straight line l has equation $y = mx + c$, where m and c are **positive** constants.

(b) Show that the x coordinates of any points of intersection of l and E satisfy the equation

$$(16 + 25m^2)x^2 + 50mcx + 25(c^2 - 16) = 0$$

(2)

Given that the line l is a tangent to E,

(c) show that $c^2 = pm^2 + q$, where p and q are constants to be found.

(3)

The line l intersects the x-axis at the point A and intersects the y-axis at the point B.

(d) Show that the area of triangle OAB, where O is the origin, is

$$\frac{25m^2 + 16}{2m}$$

(3)

(e) Find the minimum area of triangle OAB.

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

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Question 7 continued

Q7

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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